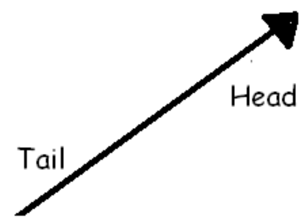
Virtual Lab – Vectors & Vector Operations

Name, Date, Period_______________________

Virtual Lab – *Vectors & Vector Operations*

<u>Set-up</u>

1. Set your calculator to degrees
2. Go to the PHET website
3. Navigate to **<u>Vector Addition</u>**
4. Click the Run Now button
5. You can drag vectors, trash them, and clear all
6. You can click and drag to change vector length and direction
7. Leave only the "sum" box unchecked



<u>Part A: 3-4-5 Triangle</u>

8. Place a vector with its tail at the origin and click-drag its head until it lies horizontally with a magnitude ($\vec{a}$) of 20
9. Notice the table at the top of the page.
    a. Note the chart at the top of the Phet Webpage.
    b. $\vec{a}$ represents the vector's length or **<u>magnitude</u>**
    c. θ represents the vector's direction, which, together with the magnitude, defines a vector
    d. **<u>a_x</u>** is the vector's **<u>X-component</u>** or its length in the x-direction
    e. **<u>a_y</u>** is the vector's **<u>Y-component</u>** or its length in the y-direction.

10. Fill out the table below for your first dragged-out vector.

$\vec{a}$	θ	a_x	a_y
20.0	0.0	20.0	0.0

11. Drag another vector $\vec{b}$ and join its tail to the first vector's head, as shown in the right-hand side. Orient it to be 15 inches long and point up. Fill out the form below for $\vec{b}$.

$\vec{b}$	θ	b_x	b_y
15	90	0.0	15

12. Click the Sum button. The vector that pops up represents the resultant of the two other vectors.

13. Drag the resultant vector to place its tail at the origin so that a right-angled triangle is formed. Click on the resultant vector and fill out the table below.

$\|s\|$	θ	s_x	s_y
25	36.9	20.0	15.0

14. Compare the s_x and s_y values to the values from questions #10 and #11. What do you notice about these values?

Comparing the four tables, we see that

$$s_x = 20.0 = 20.0 + 0 = a_x + b_x \quad \Rightarrow \quad s_x = a_x + b_x$$
$$s_y = 15.0 = 0.0 + 15.0 = a_y + b_y \quad \Rightarrow \quad s_y = a_y + b_y$$

We see that the x-components of vectors $\vec{a}$ and $\vec{b}$ add up to the x-component of their 'sum vector'.

Also, the y-components of vectors $\vec{a}$ and $\vec{b}$ add up to the y-component of their 'sum vector' $\vec{s}$.

Part B: Single Vector, Magnitude 50

15. Click the 'Clear All' button to clear the screen. Create a vector $\vec{a}$ with a_x of 20 and a_y of 15. Fill out the table below.

$\vec{a}$	θ	a_x	a_y
25	36.9	20.0	15.0

16. How do the values compare to the resultant vector in Question 13?

The vector $\vec{a}$ created here is the same vector as $\vec{s}$ created earlier: they have the same magnitude and direction, as well as the same values for the x-components and the same values for the y-components.

17. Now, hit the  **button to visualize the vector in terms of its vertical and horizontal components.**

18. Alter the vector so that a_x =15 and a_y =30 (we went to the maximum possible value of 25**). Fill out the table below for the vector.**

$\vec{a}$	θ	a_x	a_y
29.2	59.0	15	25

19. Has this vector's magnitude changed compared to Number 15? If yes, how?

The magnitude has changed. The new vector is longer than the initial vector.

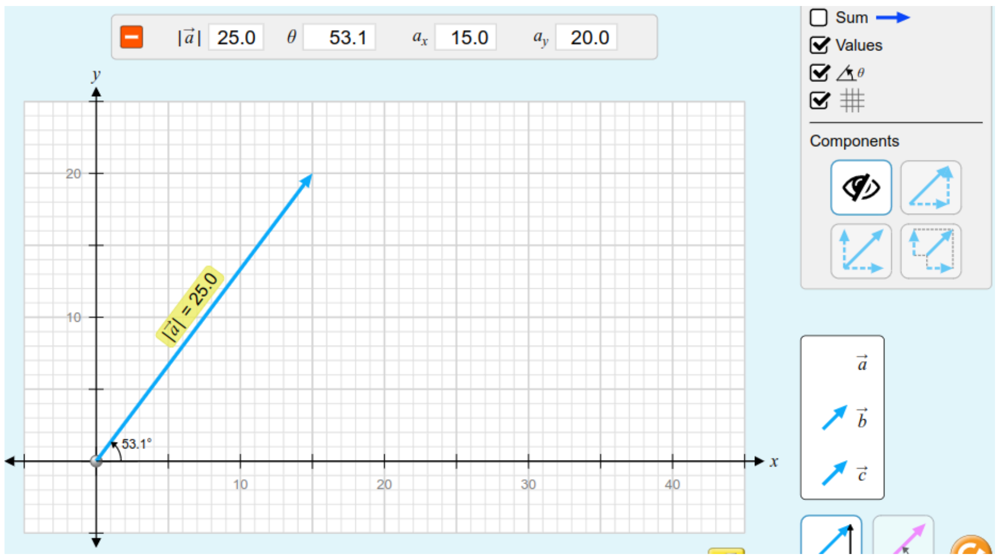
20. Has this vector's <u>direction</u> changed compared to Number 15? If yes, how?

The direction has changed. The value of the angle has increased from 36.9^o to 59^o. The new vector now points upwards more than it does eastwards, compared to Number 15.

21. **Adjust the vector's direction and magnitude until it points in a different direction and has a magnitude of 25. Fill the table below and show the vector.**

$\vec{a}$	θ	a_x	a_y
25	53.1	15	20

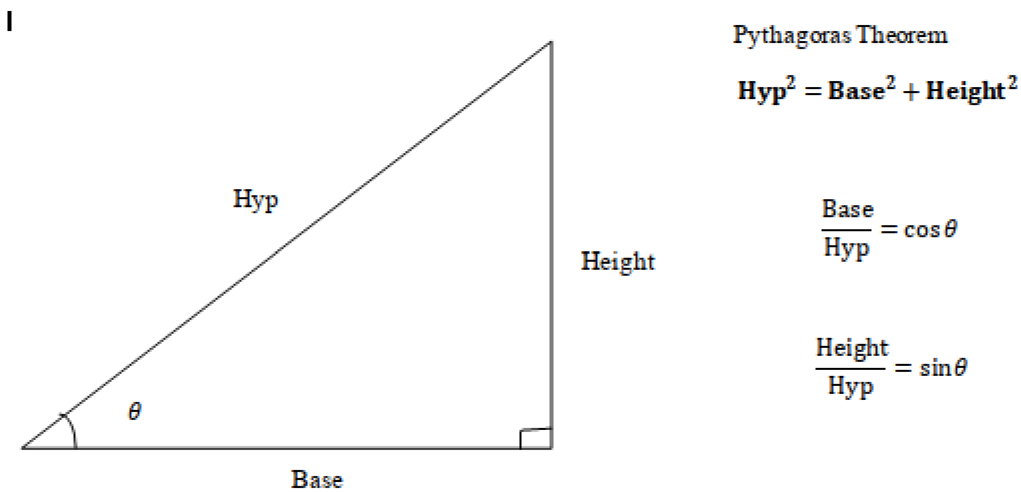
The vector is shown in the image below



22. We can imagine a right-angled triangle from the vector.

a. Use Pythagoras theorem to show that $|\vec{a}|^2 = a_x^2 + a_y^2$.

The Pythagoras theorem is stated in the figure below



Comparing to our vector figure, we see that $|\vec{a}|$ is the hypotenuse Hyp, a_x is the Base and a_y is the Height. Hence, we should have that $|\vec{a}|^2 = a_x^2 + a_y^2$. We test the equality as follows.

$|\vec{a}|^2 = 25^2 = 625$ $\Rightarrow$ $|\vec{a}|^2 = 625$

$a_x^{\ 2} + a_y^{\ 2} = 15^2 + 20^2 = 625$ $\Rightarrow$ $a_x^{\ 2} + a_y^{\ 2} = 625$

The calculations confirm that $|\vec{a}|^2 = a_x^{\ 2} + a_y^{\ 2}$.

b. **Show that $a_x = |\vec{a}| \cos \theta$.**

$\theta = 25 \cos cos\, 53.1^0 = 25 \times 0.6004 = 15.01$ $\Rightarrow$ $\theta = 15.0$

From our vector, $a_x = 15$ $\Rightarrow$ $a_x = 15$

Hence, the calculations show that $a_x = |\vec{a}| \cos \theta$.

c. **Show that $a_y = |\vec{a}| \sin \theta$.**

$|\vec{a}| \sin sin\, \theta = 25 \sin sin\, 53.1^o = 25 \times 0.7996 =$ $\Rightarrow$ $|\vec{a}| \sin sin\, \theta \approx 20$

From our vector, $a_y = 20$ $\Rightarrow$ $a_y = 20$

Hence, the calculation shows that $a_y = |\vec{a}| \sin \theta$.

23. Now think of a vector with a magnitude of $|\vec{a}| = 28$ and an angle of $\theta = 45^o$.

a. **Determine a_x and a_y, showing your work**

$$a_x = 45^0 = 28 \times 0.7071 = 19.8$$

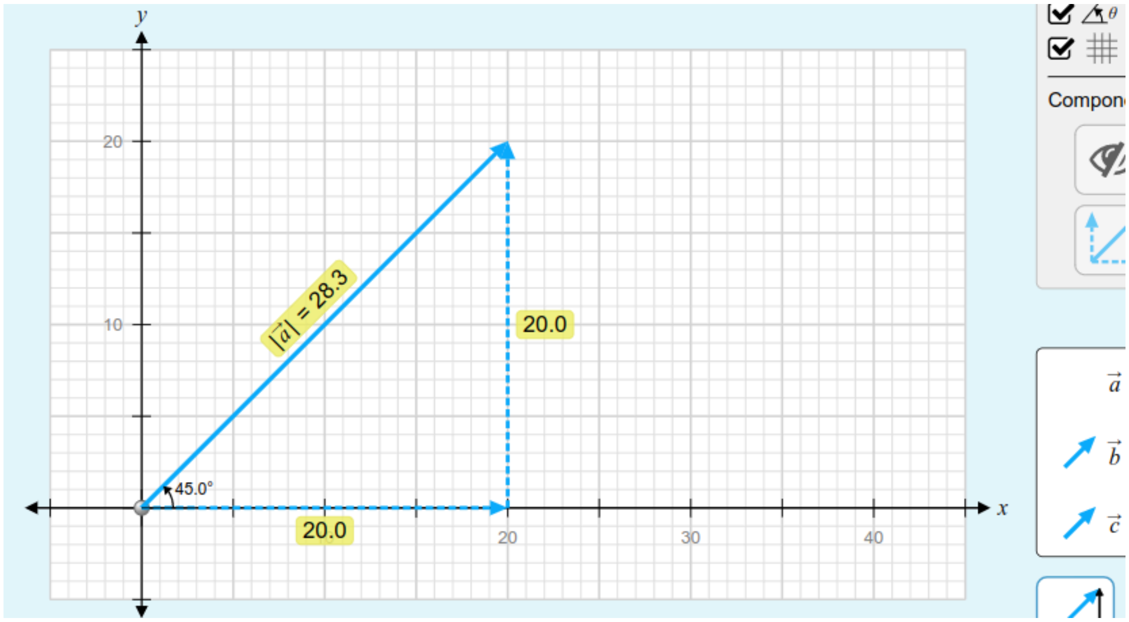
$$a_x = 19.8$$

$$a_y = |\vec{a}| \sin sin\, \theta = 28 \times \sin sin\, 45^o = 28 \times 0.7171 = 19.8$$

$$a_y = 19.8$$

b. **Construct the vector to check your answer.**

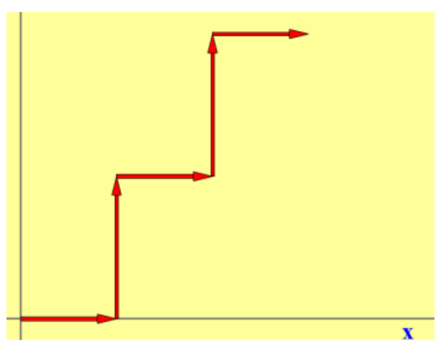
We checked the answer by constructing the vector at https://phet.colorado.edu/sims/html/vector-addition/latest/vector-addition_en.html. The platform does not allow decimal places for components, so we rounded off the figures from 23(a) and (b) to $a_x = 20$ and $a_y = 20$. The calculations were confirmed.



<u>Part C – Several Vectors</u> **(Switch to the "Lab" version of the sim)**

24. Go to the 'Lab' part of the simulation and create five same-color vectors as shown below.

Each horizontal vector should have a magnitude of 5 and each vertical a magnitude of 10.



25. Check the 'Sum' box and fill in the table below for the resultant vector

$\vec{s}$	θ	s_x	s_y
25	53.1	15	20

26. You can track the vectors by creating a table. Complete the table below and add the columns to get the sums.

Vector #	v_x	v_y
1	5	0
2	0	10
3	5	0
4	0	10
5	5	0
SUM	15	20

27. How do the sums of v_x and the sums of v_y from the previous table compare to the s_x and s_y values from question number 25?

The sum of the v_x values of all the vectors equals the s_x value of the resultant. Also, the sum of v_y values of all vectors equals the s_y value of the resultant vector.

28. **Use Pythagoras theorem to determine $|\vec{s}|$ value. Compare this value to the $|\vec{s}|$ value from question #25.**

$$s^2 = s_x^2 + s_y^2 \quad \Rightarrow \sqrt{s} = \sqrt{s_x^2 + s_y^2}$$

$$|\vec{s}| = \sqrt{s_x^2 + s_y^2} = \sqrt{15^2 + 20^2} = \sqrt{625} = 25$$

$$|\vec{s}| = 25$$

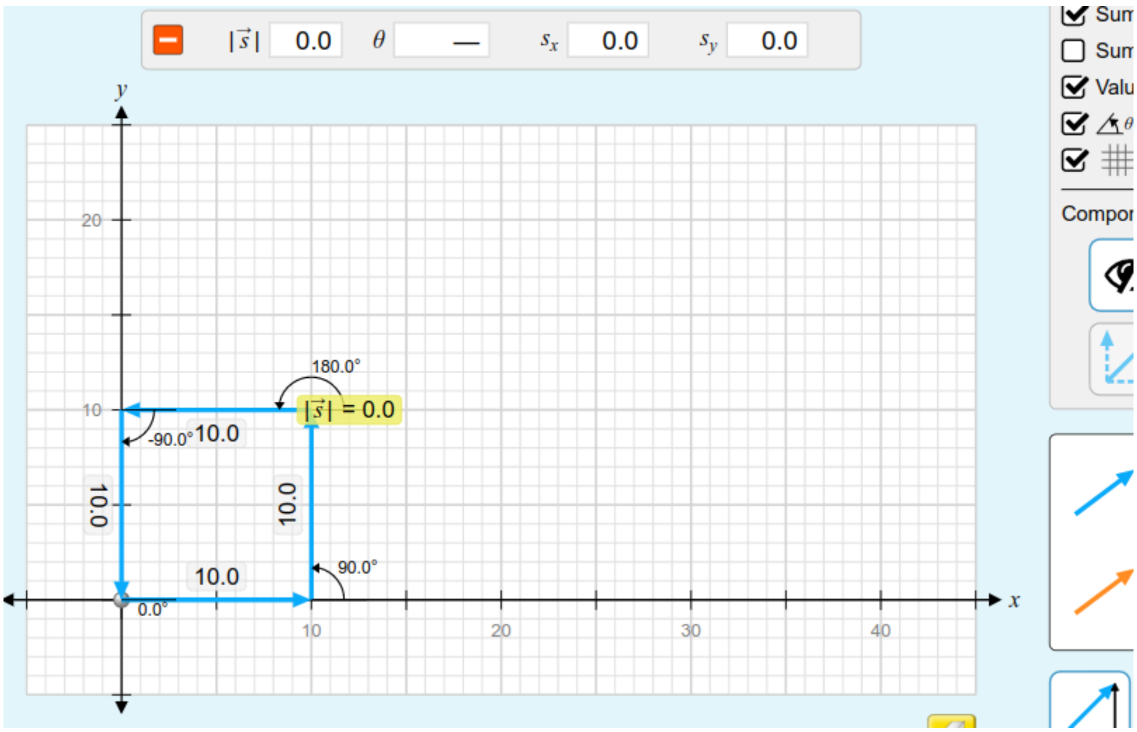
The value equals the $|\vec{s}|$ value from question 25

29. Construct the following vectors and add them "head-to-tail."

- $\vec{v} = 10, \theta = 0°$ (start this one at the origin)

- $\vec{v}$ = 10, θ = 90º
- $\vec{v}$ = 10, θ = 180º (or -180º)
- $\vec{v}$ = 10, θ = 270º (or -90º)

The vector constructed is shown below

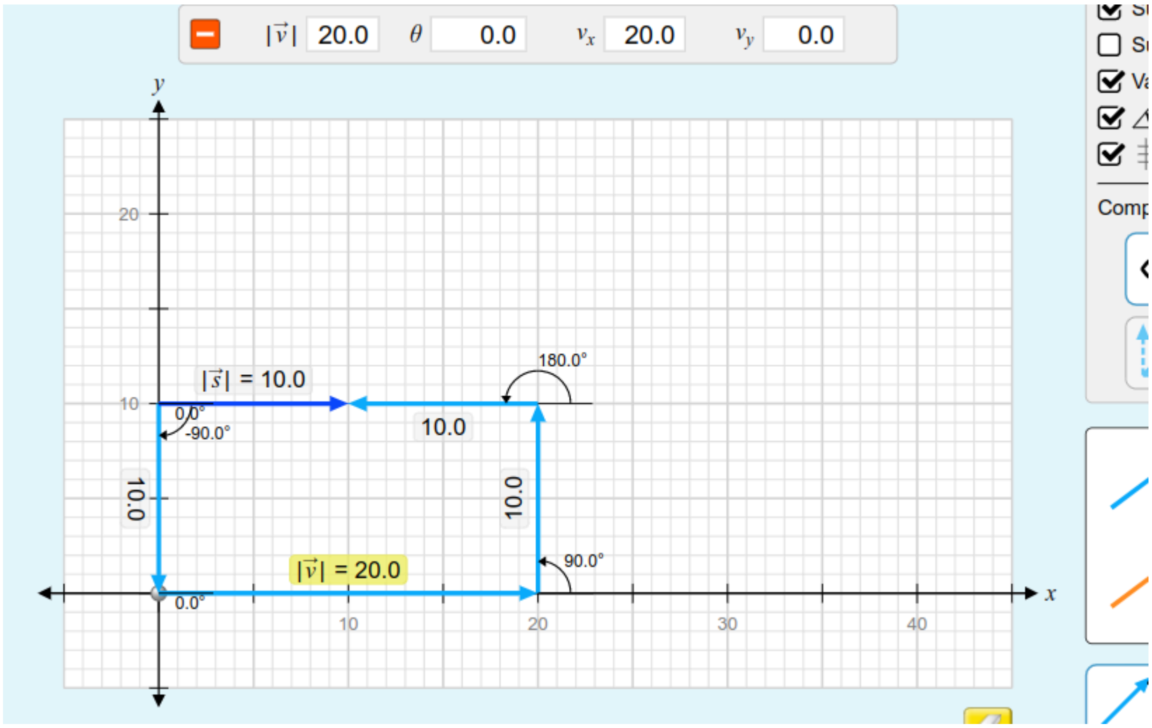


30. What is the sum (or resultant) of these vectors?

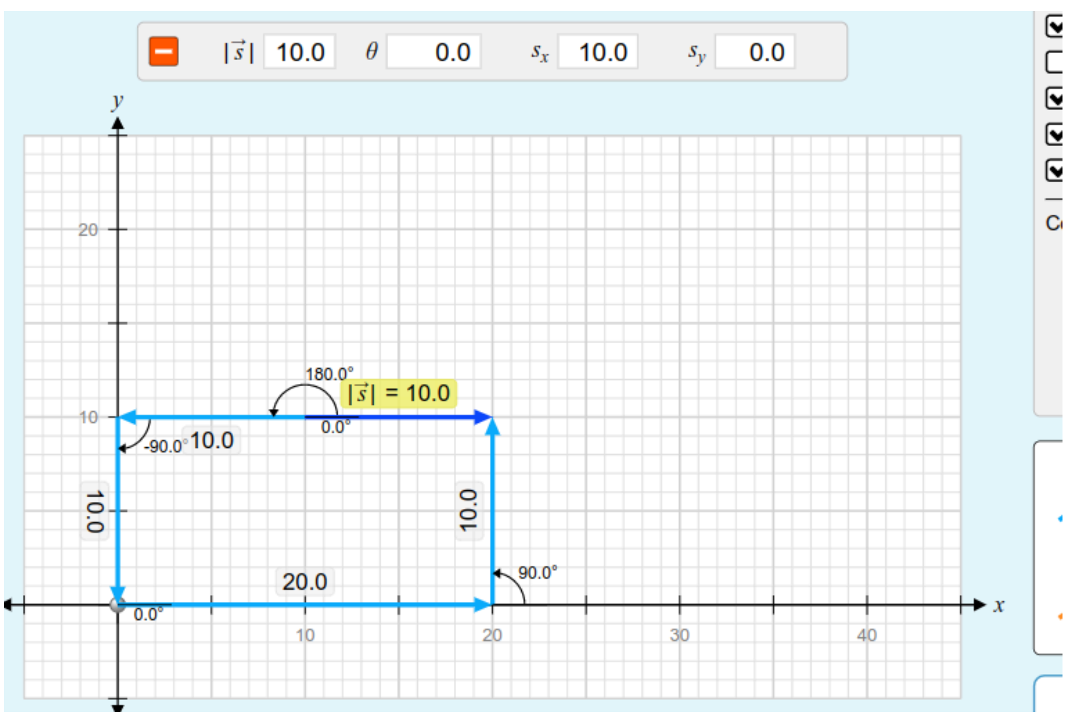
$\vec{s}$	θ	s_x	s_y
0	---- (the vector is a point, no direction)	0.0	0.0

31. What is the sum of the vectors if the first vector is 20 rather than 10?

It happens that two possible resultant vectors depending on how you arrange the vectors. The **first arrangement** is as shown in the figure below



The **second arrangement** is as shown in the figure below.



In both cases, the values for the resultant vector are as shown in the table below

$\vec{s}$	θ	s_x	s_y
10	0.0	10	0.0

32. Answer extension questions.

Extension Questions

1. A student, following instructions on her treasure map, starts at the origin and walks the following routes:

   □18 meters North (θ = 90º)

   □5 meters West (θ = 180º)

   □9 meters South (θ = 270º or -90º)

   □17 meters East (θ = 0º)

   a. **Fill out the chart below, which represents the horizontal and vertical components of the routes. Also, determine the X and Y sums.**

Vector #	v_x	v_y
1	0	18
2	-5	0
3	0	-9
4	17	0
SUM	12	9

b. **After the student has finished walking, what is her horizontal displacement? (v_x sum)**

Horizontal displacement $v_{x,sum} = 0 + (-5) + 0 + 17 = 12$

$$v_{x,sum} = 12$$

c. **What is her vertical displacement? (v_y sum)**

Vertical displacement $v_{y,sum} = 18 + 0 + (-9) + 0 = 9$

$$v_{y,sum} = 9$$

d. **Using the Pythagorean Theorem and your answers from (b) and (c), how far is she from the origin? (In other words, what is her resultant |R|?)**

Her distance from the origin,

$$|R| = \sqrt{v_x^2 + v_y^2} = \sqrt{12^2 + 9^2} = \sqrt{225} = 15$$

e. **Using SOHCAHTOA, what is her direction, as measured from the origin? (In other words, what is θ?)**

Using $a_x =$,

$$\theta = \left(\frac{a_x}{|R|}\right) = \left(\frac{12}{15}\right) = (0.8) = 36.9^o$$
$$\theta = 36.9^o$$

2. **A model airplane is flying North with a velocity of 15 m/s. A strong wind is blowing East at 12 m/s.**

a. **What is the airplane's resultant *__speed__* (magnitude of velocity vector)?**

The resultant speed is given by

$$|v| = \sqrt{v_x^2 + v_y^2} = \sqrt{12^2 + 15^2} = \sqrt{369} = 19.2m/s$$
$$|v| = 19.2m/s$$

b. **What is the airplane's direction?**

$$\theta = \left(\frac{v_x}{|v|}\right) = (\frac{12}{19.2}) \quad = (0.625) \quad = 51.3^o$$

$$\theta = 51.3^o$$

Conclusion: In 5 – 6 Sentences, how would you explain what you learned to a friend

A vector can be represented with an arrow showing its direction and a length indicating its magnitude. It can be decomposed into a horizontal (x-component) and vertical (y-component), forming what looks like a right-angled triangle with the angle θ between the two components. The sum of the squares of the two components gives the square of the magnitude of the vector. We can use trigonometric relations (SOHCAHTOA and Pythagorean Theorem) between the magnitude, the horizontal component, the vertical component, and the angle θ, to find a missing feature of the vector. To find the horizontal/vertical component of a resultant vector, simply add the horizontal/vertical components of the vectors that were combined into that resultant vector. Finally, two or more vectors can have the same magnitudes but point in different directions.